Author's reply on

Engwerda, J.C.

Published in:
IEEE Transactions on Automatic Control

Publication date:
1992

Citation for published version (APA):
Engwerda, J. C. (1992). Author's reply on: Comments on Stabilizability and detectability of discrete-time, time-varying systems. *IEEE Transactions on Automatic Control*, *37*(8), 1275.

exponentially stable filter. However, it is not indicated where the original proof in [2] is incorrect. Even more so, Counterexample 4 in the paper[1] does not contradict "Corollary 3," i.e., Anderson and Moore's result.

To be precise, with reference to the symbols of the paper[1], the characterization of uniform detectability provided in [2] requires that there exist $s, t \geq 0$ and d, b with $0 \leq d < 1$, $0 < b < \infty$, such that, whenever, for some v and k, $\|A(k+t, k)v\| \geq d\|v\|$, then $v'W[k+s, k]W[k+s, k]'v \geq bv'v$. Contrary to the author's claim, this implication is not met with by the system given in Counterexample 4. Indeed, taking $s = 0$, $t = 1$, $d = 3/4$, $b = 3/64$, as suggested in the paper[1], for $k = 0$, inequality $\|A(k+t, k)v\| \geq d\|v\|$ is equivalent to $v_1^2 \geq 9(v_1^2 + v_2^2)/16^2$, whereas inequality $v'W[k+s, k]W[k+s, k]'v \geq bv'v$ is equivalent to $v_2^2 \geq 3(v_1^2 + v_2^2)/64$. By taking $v_1 = 1$ and $v_2 = 0$, it is apparent that the second inequality is *not* implied by the first one.

Actually, it can be shown that the system in Counterexample 4 does not satisfy Anderson and Moore's detectability definition with *any* choice of s, t, d, b. Indeed, the system of this counterexample is periodic, so that it can be decomposed into reconstructible and unreconstructible parts [1], [3], [4]. Consider the pair $\hat{A}(k) = T(k+1)A(k)T(k)^{-1}$, $\hat{C}(k) = C(k)T(k)^{-1}$ that is obtained from the pair $(A(k), C(k))$ of Counterexample 4 by applying the change of coordinates $\hat{x}(k) = T(k)x(k)$. Letting

$$T(2k) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad T(2k+1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad k = 0, 1, 2, \cdots,$$

we have

$$\hat{A}(2k) = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}, \quad \hat{A}(2k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\hat{C}(2k) = [1 \ \ 0], \quad \hat{C}(2k+1) = [0 \ \ 0], \quad k = 0, 1, 2, \cdots.$$

Note that the unreconstructible part (corresponding to the second state-variable in the new basis) is unstable. Therefore, it is not difficult to verify that there exists no choice of s, t, d, b such that the pair $(\hat{A}(t), \hat{C}(t))$ satisfies Anderson and Moore's uniform detectability definition.

In conclusion, since the system in Counterexample 4 is not uniformly detectable, the impossibility of finding an exponentially stable filter does not contradict the results given in [2] and, indeed, corroborates them.

References

[1] S. Bittanti and P. Bolzern, "On the structure theory of discrete-time linear systems," *Int. J. Syst. Sci.*, vol. 17, pp. 33–47, 1986.

[2] B. D. O. Anderson and J. B. Moore, "Detectability and stabilizability of time-varying discrete-time linear systems," *SIAM J. Contr. Optimiz.*, vol. 19, pp. 20–32, 1981.

[3] S. Bittanti and P. Bolzern, "Can the Kalman canonical decomposition be performed for a discrete-time linear periodic system?," in *Proc. 1° Congresso Latino Americano de Automatica*, Campina Grande, Brazil, 1984, pp. 449–453.

[4] O. M. Grasselli, "A canonical decomposition of linear periodic discrete-time systems," *Int. J. Contr.*, vol. 40, pp. 201–214, 1984.

Author's Reply[2]

J. C. Engwerda

I was very surprised to learn from [2] that Counterexample 4 in the paper[1] does not work. I would like to thank the authors for their careful reading of my paper.

In fact, I constructed this example by dualizing the next system (see [2, lemma 2.1]

$$A(2k) = \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix};\ B(2k) = \begin{pmatrix} o \\ 1 \end{pmatrix};\ A(2k+1) = \begin{pmatrix} 0 & 0 \\ 0.5 & 0 \end{pmatrix};$$

$$B(2k+1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},\ k = 0, 1, 2, \cdots.$$

This system is uniformly stabilizable in the sense defined by Anderson and Moore in [1] (take $s = 0$, $t = 1$, $d = \sqrt{(3/4)}$, $b = (3/64)$,) whereas if we consider the initial state $x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ at time $k = 0$, we see that for any sequence $F(\cdot)$, $x(k+1) = (A(k) - B(k)F(k))x(k)$; $x(0) = x$, is not stable.

So, it is clear that there is something wrong with Definition 2.2 of uniform stabilizability in [1]. As pointed out by an anonymous referee, this definition probably contains a misprint: The state transition matrix $\hat{\phi}$ in (2.5) needs a transpose. Without the insertion of the transpose, the definition is not the dual of that for uniform detectability; with the transpose, the definition is the dual, and then and only then is Lemma 2.1 valid.

Another comment made by Bittanti *et al.* in [2], concerns the notions of controllability Gramian, observability Gramian, and the stability I introduced in notation (0) and definition (1) in the paper.[1]

I introduced these notions just for systems for time-invariant order. The generalization of these notions for systems for time-varying order is, however, so straightforward that I forgot to mention this (as I saw it after rereading my paper).

References

[1] B. D. O. Anderson and J. B. Moore, Detectability and Stabilizability of time-varying discrete-time linear systems, *SIAM J. Contr. Optimiz.* vol. 19, no. 1, pp. 20–32, 1981.

[2] S. Bittanti, P. Bolzern, and G. de Nicolao, "Comments on 'Stabilizability and detectability of discrete-time, time-varying systems,'" this issue, pp. 1274–1275.

[2] Manuscript received December 7, 1990; revised May 10, 1991.
The author is with Tilburg University, Tilburg, The Netherlands.
IEEE Log Number 9201341.